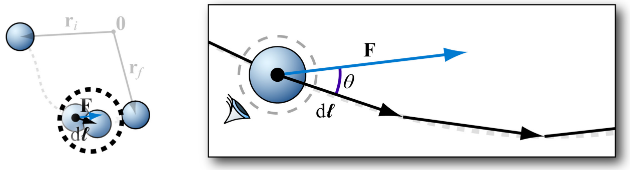
Spatial integration (for AP Physics C Mechanics)

Vector line integral | **Mass density** | **Integrals over distributions**

Neatly and graphically represent situation(s)

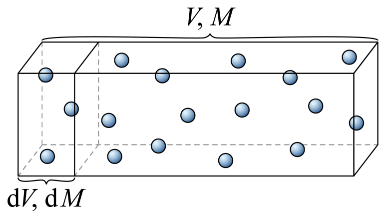
Vector line integral

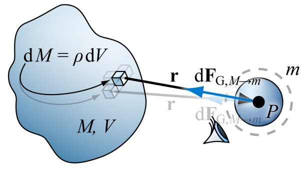
1. Draw a large diagram of the path along which the integral is to be computed.

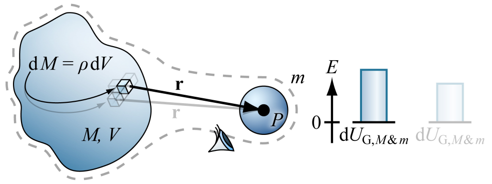


Mass density / Integrals over distributions

1. Draw a large diagram of the distribution to be integrated over.
2. Draw a reference position, axis of rotation, and/or point of observation P.







Graphically represent quantities and their relationships

2./3. Check whether the physical system exhibits symmetry(ies) permitting use of simplifying coordinate system(s).

Vector line integral

3. If needed, draw a signed coordinate system with a clear origin.
4. Draw a differential displacement $d\vec{\boldsymbol{\ell}}$. Label this differential displacement with an expression for its length $d\ell$ (possibly in terms of a coordinate system).
5. Draw the vector $\vec{\mathbf{F}}$ originating from the tail of the displacement vector.
6. Label the vector $\vec{\mathbf{F}}$ with an expression for its magnitude $|\vec{\mathbf{F}}|$ (possibly in terms of a coordinate system).
7. Draw the angle θ between the vector $\vec{\mathbf{F}}$ and $d\vec{\boldsymbol{\ell}}$.

Mass density / Integrals over distributions

4. If needed, draw a signed coordinate system with a clear origin.
5. Draw a differential element dX of the distribution, labeled in terms of a density coefficient expression and a differential geometric element (examples include $dM = \lambda\, d\ell$, $dM = \sigma\, dA$, $dM = \rho\, dV$, etc).
6. Draw and label the displacement vector $\vec{\mathbf{r}}$ from the reference position or axis of rotation to the differential element dX and/or from the differential element dX to the point of observation P.

Identify relevant allowed starting point (in)equation(s)

Vector line integral

$$Y = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}} = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \left(|\vec{\mathbf{F}}| \cos\theta\right) d\ell$$

8. Use geometry to find a formula or numerical value for the measure of the angle θ.

Mass density

$$dM = \rho\, dV$$

For a uniform mass density, $\frac{dM}{M} = \frac{dV}{V}$.

Integrals over distributions

$$Y = \int_X f(\vec{\mathbf{r}})\, dX$$

Y – output at observation point

$f(\mathbf{r})$ – expression that depends on spatial relationship between differential element of distribution and observation point

dX – differential portion of distribution

7. Use geometry to find a formula or numerical value for the length r of the displacement $\vec{\mathbf{r}}$.

Use numbered steps to show REASoNing

Communicate